

## Math 31 Sequences & Series (Part II)

**Due: Tuesday May 5<sup>th</sup> at 11:59 PM**

### Instructions

- Complete the following exercises on separate sheets of paper. Scan your solutions and upload a PDF document. The file should have the following naming convention:

“Last Name First Name Assignment Name.pdf”

“Albright Charles Chapter 11 Assignment Part 2.pdf”

- Make sure your pages are numbered in the lower right-hand corner.
- Make sure each page has your full name and the name of the assignment in the upper right-hand corner of each page.
- **Note:** You do not need to include this page in your solutions.

### Solutions

- Because of the unique circumstances of our situation, take special care with your solutions. Make sure they are complete, organized, clear and thorough. Error on the side explaining too much.
- Your final answer should be simplified and exact.

Graphs should be clear, legible and labeled.

## Math 31 Sequences & Series (Part II)

### Series

1. Determine whether the following series converge or diverge with proof.

a.  $\sum_{k=2}^{\infty} \sqrt{k^2 + 1} / \sqrt{k^5 + k - 2}$

e.  $\sum_{k=2}^{\infty} k / [(1 + k^2) \ln(1 + k^2)]$

b.  $\sum_{k=1}^{\infty} (\sqrt{k} - \sqrt{k-1})^k$

f.  $\sum_{k=1}^{\infty} \sqrt{k!} / k^k$

c.  $\sum_{n=1}^{\infty} n \sin(1/n)$

g.  $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$

d.  $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k} - 1}{k(\sqrt{k} + 1)}$

h.  $\sum_{k=1}^{\infty} 1/(k^2 + 3k + 2)$

2. Determine if the following series converges conditionally or absolutely.

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdot \dots \cdot (3n + 2)}$$

3. Determine if the following series converges conditionally or absolutely.

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi/6)}{1 + n\sqrt{n}}$$

## Math 31 Sequences & Series (Part II)

### Power Series

4. Find McLaurin series for the function  $f$  defined by  $f(x) = \frac{1}{(1-x)^4}$  using two different techniques.

5. Find a power series representation of the function  $f$  defined by

$$f(x) = \frac{1}{(1-x)^4} \text{ centered at } a = 2.$$

6. Find the Taylor series for  $f(x) = \ln(2+x)$  with center  $a = -1$ . Find the interval of convergence.

7. Find a power series representation and determine the interval of convergence  $y = \operatorname{sech} x$  at  $a = \ln(2)$ .

8. Use a power series representation to calculate the following limits and integrals:

a.  $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

c.  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$

b.  $\int \sqrt{1+x^3} dx$

d.  $\int \arctan(x^2) dx$

9. Approximate  $f$  by a Taylor polynomial with degree  $n$  at the number  $a$ . Use Taylor's Inequality to estimate the accuracy of the approximation of  $f$  when  $x$  lies in the given interval.

a.  $f(x) = \ln(1+2x), \quad a = 1, \quad n = 3, \quad 0.5 \leq x \leq 1.5$

b.  $f(x) = e^{x^2}, \quad a = 0, \quad n = 3, \quad 0 \leq x \leq 0.1$