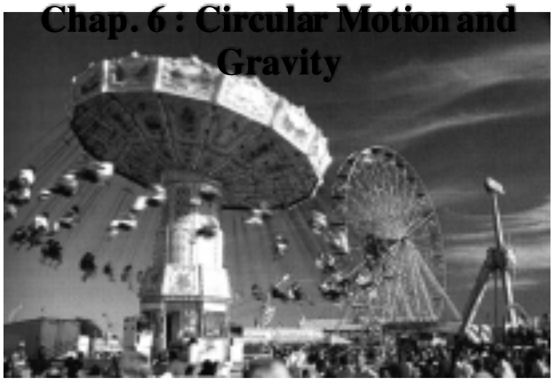
Chap. 6 : Circular Motion and Gravity



Major Topics

Uniform Circular Motion

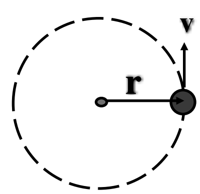
Centripetal Acceleration

Gravitation

Circular Motion

Objects moving in a circular path are accelerating.

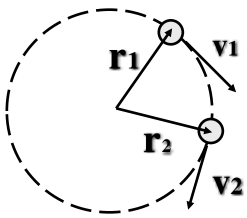
This is true even if their speed and circular radius are constant.



r = constant   v = constant

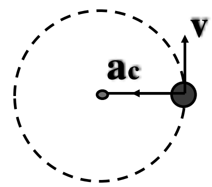
Uniform Circular Motion

Why is this true?



The magnitude is constant but the direction is changing.

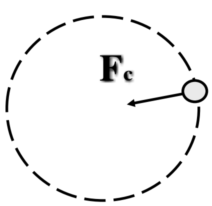
The Centripetal Acceleration



Magnitude: v^2/r

Direction: radially inward perpendicular to **v**

Centripetal Force



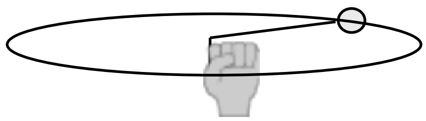
$$F_{net} = ma = ma_c = mv^2/r$$

What causes centripetal forces?

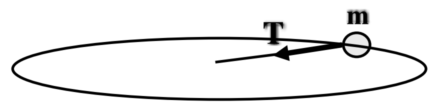
- This is not a "new force" to consider.
- The previous slide shows that it is always the *net force* .
- It will be the result of all the forces acting on the system.
- Results from T, mg , n , F_a , f_s , f_k.

Example Problems

1. A ball is whirled around a horizontal circle on the end of a string of fixed length at a constant speed. Explain the motion of the ball? Ignore any effects due to gravity.



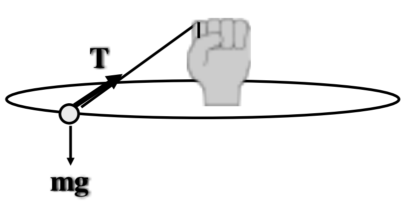
Freebody Analysis

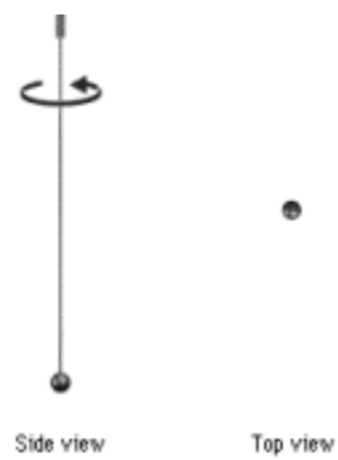


$$\Sigma F = ma$$

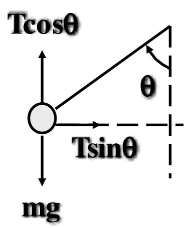
$$T = \frac{mv^2}{r}$$

What happens if gravity is not ignored?



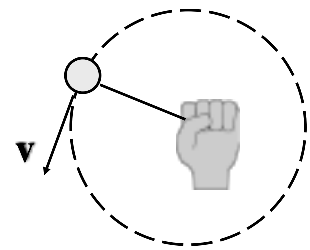


Bigger v means bigger θ

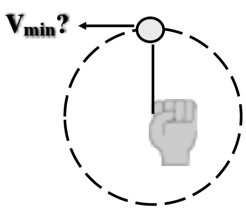


The more horizontal the ball is spun (larger θ), the bigger v must be.

2. Explain what happens when a ball is whirled around on the end of a string vertically ?

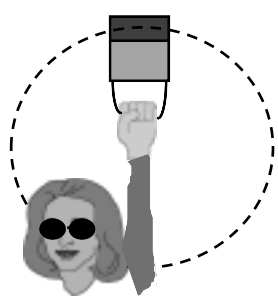


2a. What is the minimum speed needed to complete the loop for a given radius ?



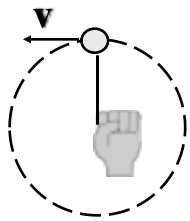
Why is this a significant question?

What is the minimum speed that one can swing a bucket of water over one's head vertically without getting wet ?

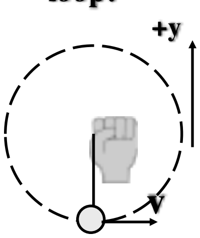


What is the minimum speed needed to keep the passengers from falling out?

Top of Loop



2b. What happens at the bottom of the loop?



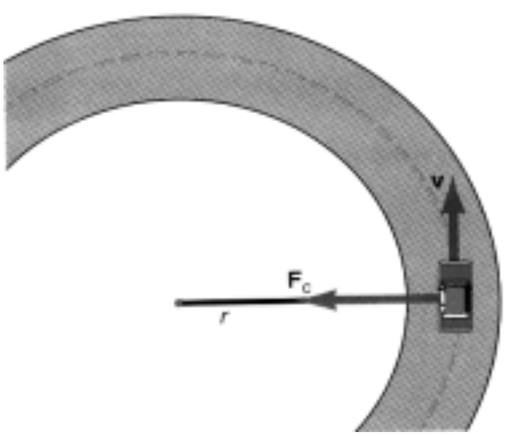
If the object is spun faster and faster, where is the string most likely to break?

Progress Check

You are riding a roller coaster which has a loop of radius 10 m. Your speed at the top of the loop is the minimum value to keep you from falling out. What is your apparent weight at the bottom of the loop if frictional effects can be ignored?

3. What is the maximum speed that a car can drive over a hill and maintain contact with the road?

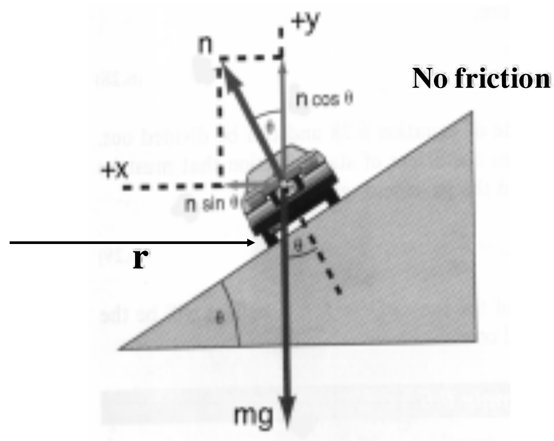
4. How fast can a car travel and make a turn without slipping?



Progress Check

A car is safely negotiating an unbanked circular turn at a speed of 21 m/s. The maximum static frictional force acts on the tires. Suddenly a wet patch in the road reduces the maximum static frictional force by a factor of three. If the car is to continue safely around the curve, to what speed must the driver slow the car?

5. **How does banking a turn help?**



General Formulas

Friction only: $v_{\max} = \sqrt{g r \mu_s}$

Banking only: $v_{\max} = \sqrt{g r \tan\theta_b}$

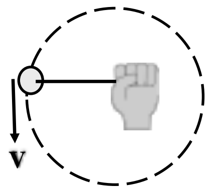
$$v_{\max} = \sqrt{g r \frac{(\mu_s \cos\theta_b + \sin\theta_b)}{(-\mu_s \sin\theta_b + \cos\theta_b)}}$$

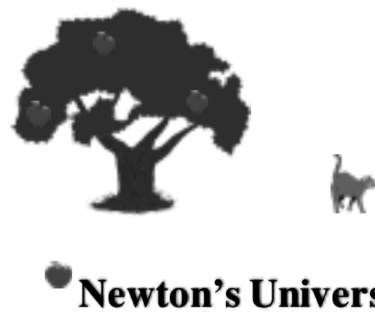
An object moving in a circle can have two accelerations!

Actually only one, $\mathbf{a}_{net}$, with two components:

$$\mathbf{a}_{net} = \mathbf{a}_T + \mathbf{a}_C$$

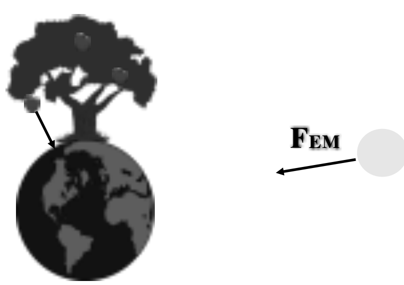
Example: A ball of mass 1 kg spinning in a vertical circle has a speed of 7 m/s at the position shown. What is the magnitude and direction of the ball's acceleration if the length of the string is 2 m?



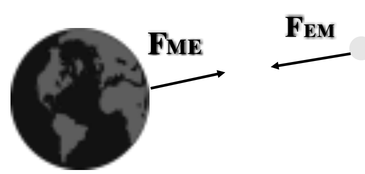


Newton's Universal Law of Gravitation

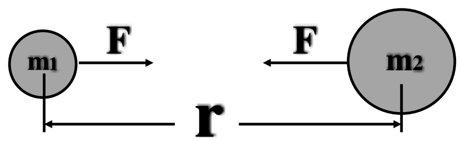
The Apple and the Moon



A 3rd Law Pair



Newton's Universal Law of Gravitation

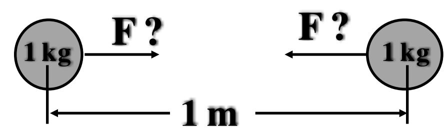


$$F = G\frac{m_1 m_2}{r^2}$$

(magnitude only)

$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

What is the force due to gravity pulling on two 1 kg masses separated by 1 m?



The Answer

- 6.67×10^{-11} N

When does F_g become important?

- When one (or more) of the masses is large!

What happens to g as we move away from the earth's surface?

$$ma = \sum F$$

$$mg = G\frac{m_E m}{r^2}$$

$$g = G\frac{m_E}{r^2}$$

Example

How far above the earth's surface is g equal to 4.9 m/s^2 (1/2 g)?

Important Facts

Gravitational forces:

1. are only attractive
2. are relatively small
3. explain weight (mg)
4. fall off as $1/r^2$
5. are "Central Forces"
6. obey the "Superposition Principle"

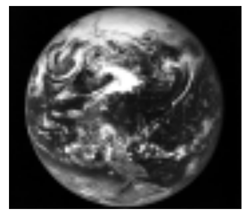
Mass & Weight

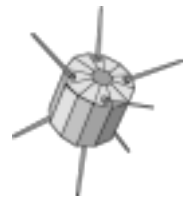
Mass:

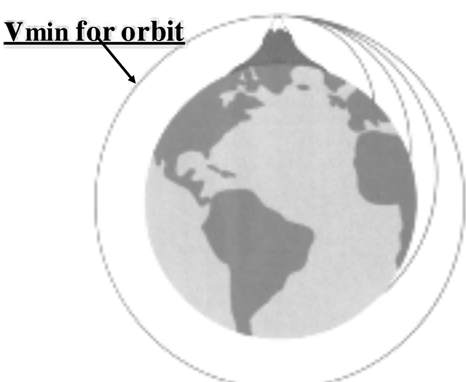
1. A measure of the amount of matter.
2. A measure of inertia.
3. A constant property independent of location.

Weight: A force due to gravity dependent on location.

Why do satellites orbit the earth ?

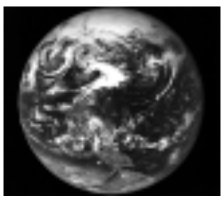






What does v_{min} depend upon?

What is the speed of a satellite that orbits the earth at two earth radii above the earth's surface?



What is the period of a satellite?

What is the height of a geosynchronous satellite above the earth's surface?

Progress Check

An artificial satellite circling the Earth completes each orbit in 90.0 minutes. What is the value of g at the location of this satellite?

How do forces act at a distance?

- **Newton's Dilemma**
- **All forces act at a distance!**

The Gravitational Field

1. "A region of space that will exert a gravitational force on a mass when placed there."
2. Space is altered by the presence of a mass.
3. The "field strength" of the source is independent of the presence of another mass and will fall off as $1/r^2$.

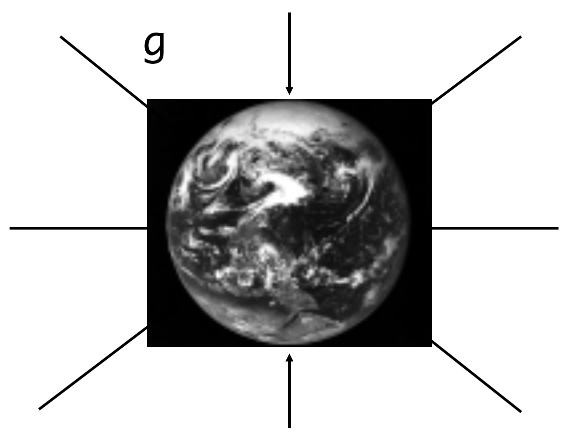
The Gravitational Field

By definition: "The gravitational field is the force per unit mass."

$$\mathbf{g} = -G\frac{m_E}{r^2}\,\mathit{r}$$

Field Representation

1. Fields are represented with lines that show both magnitude and direction.
2. The direction of the force is the same as the direction of the line at a particular location.
3. The spacing of the lines gives the relative field strength.
4. The closer the lines, the stronger the field.
5. Field lines can never cross.



Gravitational Potential Energy

$\Delta U = mgh$ (at Earth's surface)

A more general treatment:

$$\Delta U = -Gm_1m_2\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Absolute U

Where is the zero gravitational U ?

At infinity !

Reference points are arbitrary, but it is more convenient to choose one where the force is zero.

$$U_0 = 0 \text{ at } r_0 = \infty$$

$$U = -GM_Em(r^{-1})$$

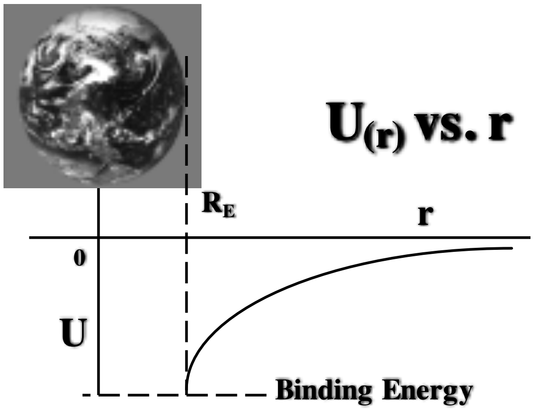
This applies when considering any two masses separated by a distance r, where $r \geq \Sigma$ radii.

This choice means that U(r) will always be negative.

Some Important Facts

1. **U_g varies as 1/r for any pair of particles.**
2. **F_g varies as $1/r^2$ for any pair of particles**
3. **U_g is negative because F_g is attractive and $U_0 = 0$ at ∞ .**
4. **An external agent must do positive work to separate a pair of particles which produces an increase in U (becomes less negative as r increases).**

$U_{(r)}$ vs. r



The Binding Energy

An energy equal to $+Gm_1m_2/r$ must be supplied to totally separate the two masses. The absolute value of the potential energy is called the "binding energy" of the system.

What happens when more than the binding energy is supplied ?

To Escape the Earth

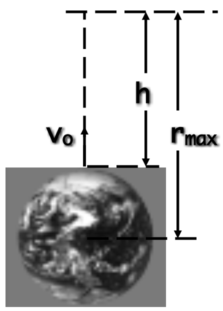
Potential Energies Are Additive

Due to the Superposition Principle:

$$U_T = \Sigma U$$

U_T is the work needed to separate the particles by an infinite distance.

Escape Speed



$E_T = K + U$

$E_T = (1/2)mv_{esc}^2 + (-GMmR_E^{-1}) = 0$

$v_{esc} = (2GM_ER_E^{-1})^{1/2}$

This result is independent of m or direction of v.

What is v_{esc} from the Earth ?

Problem 41c from Text

What is the minimum energy needed to place a 200 kg satellite into an orbit at 200 km above the surface of the earth? Assume that the satellite is launched from the equator, that air friction is negligible and that the orbit is circular.

Chapter 9: Major Topics

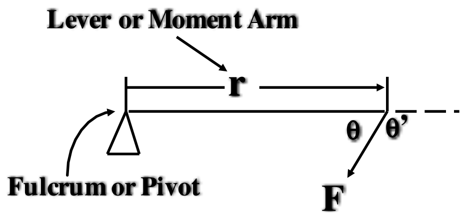
Torque
Center of Gravity
Equilibrium
Angular Momentum (8.5, 8.7)

Torque

Torques Cause Rotations

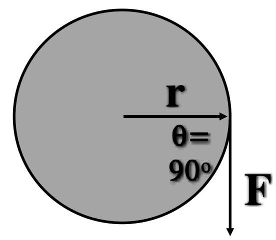
These two situations are identical except for the point at which the force acts, and the results are quite different.

Torque Defined



$$\tau\tau = rF\sin\theta = rF\sin\theta'$$

Maximum Torque



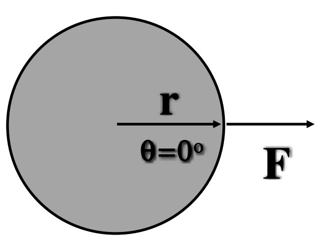
$$\tau = rF\sin\theta$$

$$\tau = rF\sin 90^o$$

$$\tau_{max} = rF$$

Maximum Rotation Acceleration

Minimum Torque



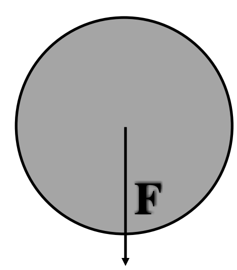
$$\tau = rF\sin\theta$$

$$\tau = rF\sin 0^o$$

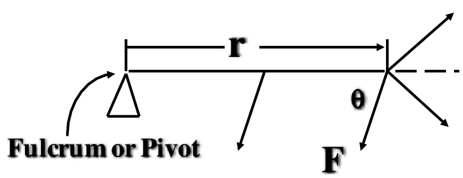
$$\tau_{min} = 0$$

No Rotational acceleration

What is the torque here ?



The Possibilities



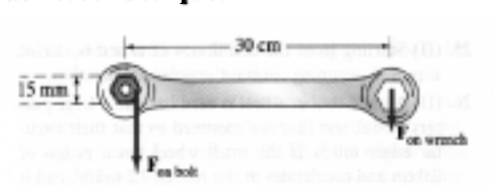
$$\tau = rF\sin\theta$$

How To Find The Torque

1. Locate the pivot.
2. Draw a line from the pivot to the point where the force is applied to define the "moment arm".
3. Draw the force vector.

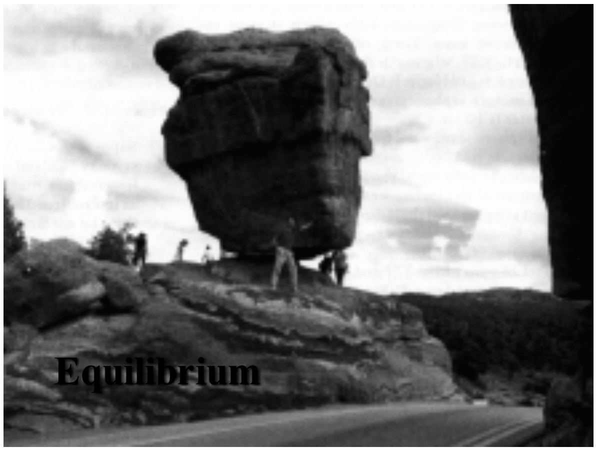
Progress Check

The bolts on the cylinder head of an engine require tightening to a torque of 80 mN. If a wrench is 30 cm long, what perpendicular force must a mechanic exert at its end to achieve this torque?



Progress Check

A square, 0.40 m on a side, is mounted so that it can rotate about an axis that passes through the center of the square. The axis is perpendicular to the plane of the square. A force of 15 N lies in this plane and is applied to the square. What is the magnitude of the maximum torque that such a force could produce?



Two Necessary and Sufficient Conditions for Equilibrium

1. $\Sigma F = 0$ (no translational a)
2. $\Sigma \tau = 0$ (no rotational a)

Demonstration Problem

A uniform meter stick is suspended at the 50 cm mark and a mass m_1 is placed at the 90 cm mark. Where would a mass m_2 have to be placed to bring this system of masses to "balance" ?

Center of Gravity

"A point in space where all the mass of a particular system *behaves* as if it is concentrated."

"It follows all the laws of physics."

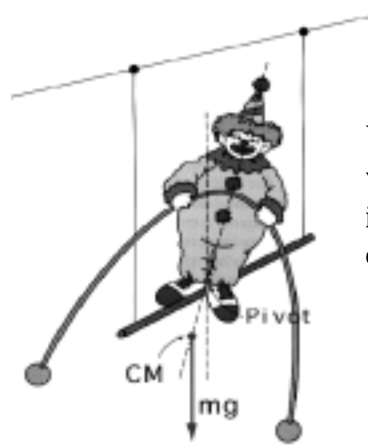
CG Quantitatively

$$R = \frac{\Sigma\, m_n r_n}{m_T}$$

$$x = \frac{\Sigma\, m_n x_n}{m_T} \qquad y = \frac{\Sigma\, m_n y_n}{m_T}$$

Example Problem

Three particles are located on the x axis at –6, 4 and 7 m with masses of 1, 2, and 3 kg respectively. Where is the CG?

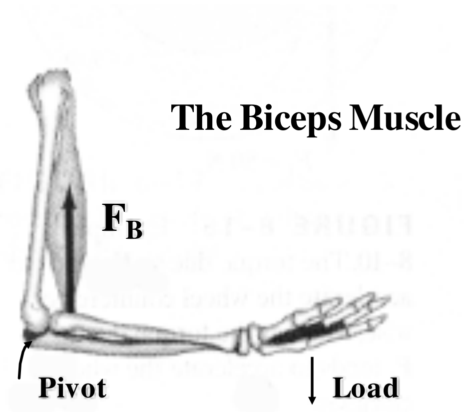


What happens when a system is not in equilibrium ?

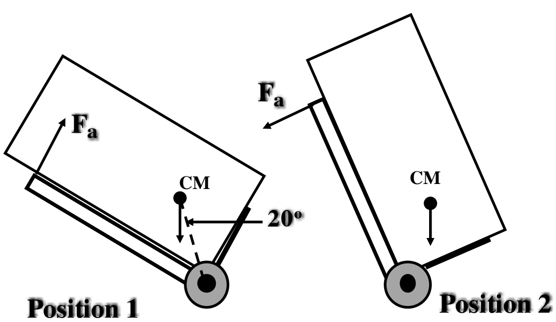
The Problem

What is the force of the biceps muscle required to hold a 25 pound barbell as shown in the next slide? The muscle acts at a distance of 3 cm from the pivot and the load is at a distance of 35 cm.

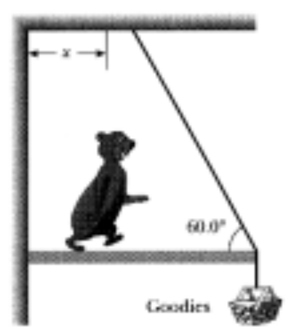
The Biceps Muscle



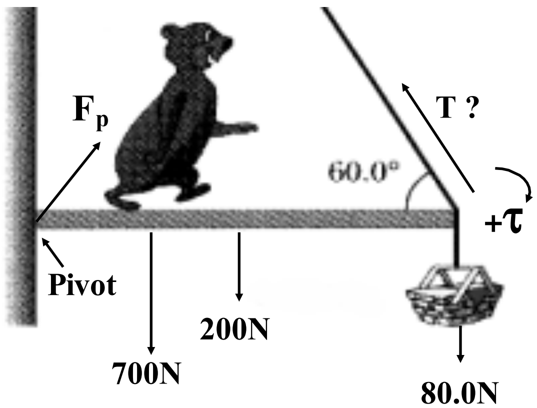
What good is a dolly?



Progress Check



This bear is hungry. He weighs 700 N, the basket weighs 80.0 N, and the uniform beam is 6.00m long and weighs 200 N. What is the tension in wire if the bear is at x = 1.00 m?

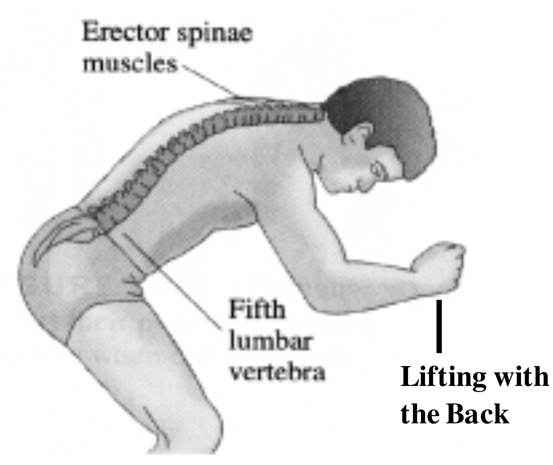


Not so fast!

What is the force exerted by the wall on the left end of the beam?
(Remember that force is a vector with both magnitude and direction.)

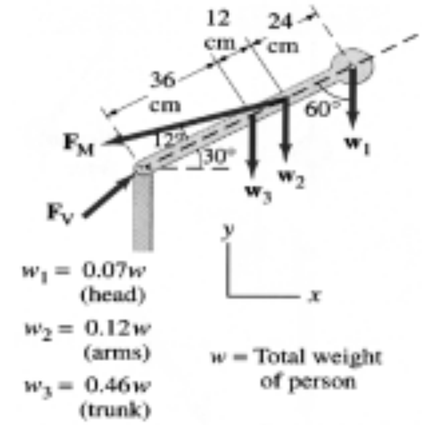
Are you tired of this bear?

How close can the bear get to the goodies before the wire breaks? The wire can withstand a maximum tension of 900 N.



Is lifting dangerous ?

Calculate the magnitude and direction of the force F_V acting on the lumbar vertebra for the following example.



It is little wonder that people have lower back problems!

The force on the lowest vertebra is 2 1/2 times the body weight. This force is transmitted from the sacral bone at the base of the spine through the fluid-filled and somewhat flexible intervertebral disc. The discs are clearly being compressed under very high forces. If a 200-lb person (90 kg) is holding 20 kg (44-lb), F_v is increased to 5w and the compression force would be 1000-lb ! How can F_v be reduced?

The Ladder Problem

A 10 kg monkey climbs up a 120 N ladder of length L as shown. The upper and lower ends of the ladder rest on frictionless surfaces. The lower end is fastened to the wall by a horizontal rope that can support a maximum tension of 110 N. What is the tension in the rope when the monkey is one third the way up the ladder?

Rotational Quantities

Quantity	Translation	Rotation
Position	x	θθ
Vel	$v_{ave} = \Delta x/\Delta t$	$\omega = \Delta\theta / \Delta t$
Accel.	$a = \Delta v/\Delta t$	$\alpha = \Delta\omega / \Delta t$
Cause	Force	Torque

Ch. 10 Rotational Motion

- Kinematics
- Dynamics
- Angular Momentum

Kinematics Example

At t = 0 a grinding wheel has an angular velocity of 24 rad/s. Then it coasts to a stop turning through 480 rad with a constant acceleration. (a) What is the angular acceleration? (b) How long did it take to stop?

Rotational Quantities

Quantity	Translation	Rotation
Position	x	θ
Vel	$v_{ave} = \Delta x/\Delta t$	$\omega = \Delta\theta / \Delta t$
Accel.	$a = \Delta v/\Delta t$	$\alpha = \Delta\omega / \Delta t$
Cause	Force	Torque
Inertia	mass	Rot. Inertia (I)

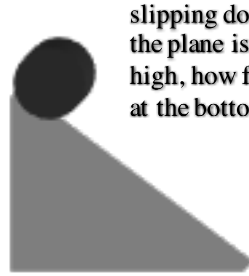
Rotational Inertia

Rotational Quantities

Quantity	Translation	Rotation
Position	x	$\theta\theta$
Vel	$v_{ave} = \Delta x / \Delta t$	$\omega = \Delta\theta / \Delta t$
Accel.	$a = \Delta v / \Delta t$	$\alpha = \Delta\omega / \Delta t$
Cause	Force	Torque
Inertia	mass	Rot. Inertia (I)
KE	$^1/_2 mv^2$	$^1/_2 I\omega^2$

Sphere on an Incline

Example



A cylinder of mass M and radius R starts from rest and rolls without slipping down an incline plane. If the plane is 1.65 m long and 1.0 m high, how fast is the cylinder moving at the bottom?

Rotational Quantities

Quantity	Translation	Rotation
Position	x	θθ
Vel	$v_{ave} = \Delta x/\Delta t$	$\omega = \Delta\theta / \Delta t$
Accel.	$a = \Delta v/\Delta t$	$\alpha = \Delta\omega / \Delta t$
Cause	Force	Torque
Inertia	mass	Rot. Inertia (I)
KE	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Momentum	$P = mv$	$L = I\omega$

Conservation of Momentum

Translational

$p_{initial} = p_{final}$:
No external forces

Rotational

$L_{inital} = L_{final}$:
No external torques

Example

A free-spinning merry-go-round is making 1 revolution every 6 s and has a moment of inertia I = 1200 kgm². A 40 kg child stands at the center and then walks toward the rim. When the child is 2 m from the center, what is the angular speed of the merry-go-round? ($I_{child} = mr^2$)