

Find the length of the curve for

$$x = \ln\left(\frac{1}{1-y}\right)$$

between 0 and $\ln(2)$.

$$\begin{aligned} \frac{dx}{dy} &= 1-y \left(\frac{0-1(-1)}{(1-y)^2} \right) \\ &= \frac{1-y}{(1-y)^2} = \frac{1}{1-y} \end{aligned}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{(1-y)^2}$$

$$L = \int_0^{\ln 2} \sqrt{1 + \frac{1}{(1-y)^2}} dy$$

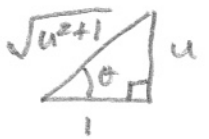
$$= \int_0^{\ln 2} \sqrt{\frac{(1-y)^2 + 1}{(1-y)^2}} dy$$

$$= \int_0^{\ln 2} \frac{\sqrt{(1-y)^2 + 1}}{1-y} dy$$

$$\begin{aligned} u &= 1-y \\ du &= -dy \end{aligned}$$

$$= - \int_0^{\ln 2} \frac{\sqrt{u^2 + 1}}{u} dy$$

$$\begin{aligned} u &= \tan \theta \\ du &= \sec^2 \theta d\theta \\ \theta &= \tan^{-1}\left(\frac{u}{1}\right) \\ \sec \theta &= \sqrt{u^2 + 1} \end{aligned}$$



$$= - \int_0^{\ln 2} \frac{\sec \theta}{\tan \theta} \sec^2 \theta d\theta$$

$$= - \int_0^{\ln 2} \frac{\sec^3 \theta}{\tan \theta} d\theta$$

$$= - \int_0^{\ln 2} \frac{1}{\cos^2 \theta \sin \theta} d\theta$$

$$\begin{aligned} \frac{\sec^3 \theta}{\tan \theta} &= \frac{\cos \theta}{\cos^3 \theta \sin \theta} \\ &= \frac{1}{\cos^2 \theta \sin \theta} \end{aligned}$$

$$= - \int_0^{\ln 2} \frac{\sec^2 \theta}{\sin \theta} d\theta$$

$$\begin{aligned} w &= \sin \theta \\ dw &= \cos \theta d\theta \end{aligned}$$

$$\begin{aligned} dv &= \sec^2 \theta d\theta \\ v &= \tan \theta \end{aligned}$$

$$= - \left[\sin \theta \tan \theta - \int_0^{\ln 2} \tan \theta \cos \theta d\theta \right]$$

$$= - \left[\sin \theta \tan \theta - \int_0^{\ln 2} \frac{\sin \theta \cos \theta}{\cos \theta} d\theta \right]$$

$$= - \left[\sin \theta \tan \theta - \int_0^{\ln 2} \sin \theta \, d\theta \right]$$

$$= - \left[\sin \theta \tan \theta - (-\cos \theta) \right]_0^{\ln 2}$$

$$= - \left[\sin \theta \tan \theta + \cos \theta \right]_0^{\ln 2}$$

$$= - \left[\frac{u}{\sqrt{u^2+1}} \cdot \frac{u}{1} + \frac{1}{\sqrt{u^2+1}} \right]_0^{\ln 2}$$

$$= - \left[\frac{u^2 + 1}{\sqrt{u^2+1}} \right]_0^{\ln 2}$$

$$= - \left[\frac{(1-y)^2 + 1}{\sqrt{(1-y)^2 + 1}} \right]_0^{\ln 2}$$

$$= - \left[\frac{1 - 2y + y^2 + 1}{\sqrt{1 - 2y + y^2 + 1}} \right]_0^{\ln 2}$$

$$= - \left[\frac{y^2 - 2y + 2}{\sqrt{y^2 - 2y + 2}} \right]_0^{\ln 2}$$

$$= - \left[\frac{(\ln 2)^2 - 2 \ln 2 + 2}{\sqrt{(\ln 2)^2 - 2 \ln 2 + 2}} - \left(\frac{2}{\sqrt{2}} \right) \right]$$

$$= - \left[\frac{(\ln 2)^2 - 2 \ln 2 + 2}{\sqrt{(\ln 2)^2 - 2 \ln 2 + 2}} - \sqrt{2} \right]$$

$$\theta = \tan^{-1} \left(\frac{u}{1} \right)$$



$$u = 1 - y$$