Quiz 7
Zed Chance

Find the length of the curve for

$$x = \ln\left(\frac{1}{1-y}\right)$$

between 0 and ln(2).

$$\frac{\partial x}{\partial y} = 1-y\left(\frac{0-1(-1)}{(1-y)^2}\right)$$

$$= \frac{1-y}{(1-y)^2} = \frac{1}{1-y}$$

$$\left(\frac{\partial x}{\partial y}\right)^2 = \frac{1}{(1-y)^2}$$

$$L = \int_0^{\ln 2} \sqrt{1 + \frac{1}{(1-y)^2}}\, dy$$

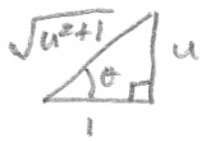
$$= \int_0^{\ln 2} \sqrt{\frac{(1-y)^2+1}{(1-y)^2}}\, dy$$

$$= \int_0^{\ln 2} \frac{\sqrt{(1-y)^2+1}}{1-y}\, dy$$

$$u = 1-y$$
$$du = -\partial y$$

$$= -\int_0^{\ln 2} \frac{\sqrt{u^2+1}}{u}\, dy$$

$$u = \tan\theta$$
$$du = \sec^2\theta\, \partial\theta$$
$$\theta = \tan^{-1}\left(\frac{u}{1}\right)$$
$$\sec\theta = \sqrt{u^2+1}$$



$$= -\int_0^{\ln 2} \frac{\sec\theta}{\tan\theta} \sec^2\theta\, \partial\theta$$

$$= -\int_0^{\ln 2} \frac{\sec^3\theta}{\tan\theta}\, \partial\theta$$

$$\frac{\sec^3\theta}{\tan\theta} = \frac{\cos\theta}{\cos^3\theta \sin\theta}$$
$$= \frac{1}{\cos^2\theta \sin\theta}$$

$$= -\int_0^{\ln 2} \frac{1}{\cos^2\theta \sin\theta}\, \partial\theta$$

$$= -\int_0^{\ln 2} \frac{\sec^2\theta}{\sin\theta}\, d\theta$$

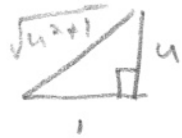
$$w = \sin\theta \qquad \partial v = \sec^2\theta\, \partial\theta$$
$$dw = \cos\theta\, \partial\theta \qquad v = \tan\theta$$

$$= -\left[\sin\theta \tan\theta - \int_0^{\ln 2} \tan\theta \cos\theta\, \partial\theta\right]$$

$$= -\left[\sin\theta \tan\theta - \int_0^{\ln 2} \frac{\sin\theta \cos\theta}{\cos\theta}\, \partial\theta\right]$$

$$= -\left[\sin\theta\tan\theta - \int_0^{\ln 2}\sin\theta\,d\theta\right]$$

$$\theta = \tan^{-1}\left(\frac{u}{1}\right)$$



$$u = 1 - y$$

$$= -\left[\sin\theta\tan\theta - (-\cos\theta)\right]_0^{\ln 2}$$

$$= -\left[\sin\theta\tan\theta + \cos\theta\right]_0^{\ln 2}$$

$$= -\left[\frac{u}{\sqrt{u^2+1}}\cdot\frac{u}{1} + \frac{1}{\sqrt{u^2+1}}\right]_0^{\ln 2}$$

$$= -\left[\frac{u^2+1}{\sqrt{u^2+1}}\right]_0^{\ln 2}$$

$$= -\left[\frac{(1-y)^2+1}{\sqrt{(1-y)^2+1}}\right]_0^{\ln 2}$$

$$= -\left[\frac{1-2y+y^2+1}{\sqrt{1-2y+y^2+1}}\right]_0^{\ln 2}$$

$$= -\left[\frac{y^2-2y+2}{\sqrt{y^2-2y+2}}\right]_0^{\ln 2}$$

$$= -\left[\frac{(\ln 2)^2 - 2\ln 2 + 2}{\sqrt{(\ln 2)^2 - 2\ln 2 + 2}} - \left(\frac{2}{\sqrt{2}}\right)\right]$$

$$\boxed{= -\left[\frac{(\ln 2)^2 - 2\ln 2 + 2}{\sqrt{(\ln 2)^2 - 2\ln 2 + 2}} - \sqrt{2}\right]}$$