Math 31 | Exam 2: Chapter 6, Chapter 8 and Chapter 9

Due: April 16th by 11:59PM

Instructions

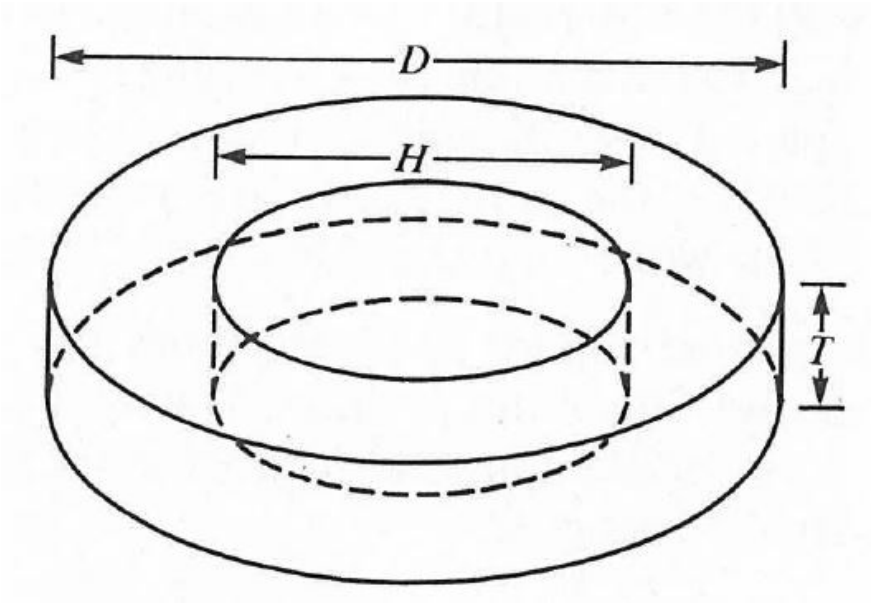
- Complete the following exercises on sperate sheets of paper. Scan your solutions and upload a PDF document. The file should have the following naming convention:

"Last Name First Name Assignment Name.pdf"
"Albright Charles Exam 2.pdf"

- Make sure your pages are numbered in the lower right-hand corner.
- Make sure each page has your full name and the name of the assignment in the upper right-hand corner of each page.
- **Note:** You do not need to include this page in your solutions.

Solutions

- Because of the unique circumstances of our situation, take special care with your solutions. Make sure they are complete, organized, clear and thorough. Error on the side explaining too much.
- Your final answer should be simplified and exact.
- Graphs should be clear, legible and labeled.

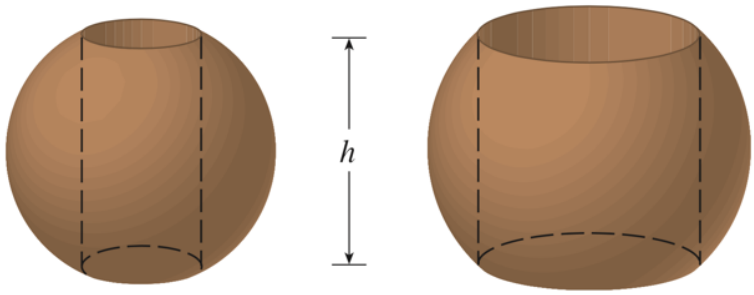


Questions

1. This equation defines a famous curve with a loop: $y^2 = x^2(x+3)$. Graph this curve and find the area inside the loop.

2. There is a line through the origin that divides the region bounded by the parabola $y = x - x^2$ and the *x*-axis into two regions with equal area. Graph these curves. What is the slope of that line?

3. Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height h, as shown in the figure. Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius r through the center of a sphere of radius R and express the answer in terms of h.



4. Find the volume common to two circular cylinders, each with radius r, if the axes of the cylinders intersect at right angles.

5. Find the volume of the curve $y = 1 + \sin(x)\cos(x)$:
   a. For $0 \le x \le \pi$, rotated around the x-axis. Draw the figure.
   b. For $-\pi \le x \le \pi$, rotated around the line $x = 5$. Draw the figure.

6. A cylindrical drum filled with water is laying on its side. (That is, the circular face of the drum would be facing you if you were looking at the drum) The drum has a radius of 1 meter and is and length 3 meters.
   a. How much work is required to pump all of the water out of the drum through a spout that is one meter above the top of the tank?
   b. How much of the water could be pumped out of the tank if you were permitted to only use half of the work from part (a).

7. A water tank extends backward 10 meters with a front face in the shape of an rectangle. The tank has a height of 5 meters and a width of 10 meters at its base. A spout is located 2 meters above the top of the tank. On the front of the tank (in the middle) is a small circular port 0.5 meters in diameter.
    a. Calculate the work to empty the tank.
    b. Calculate the force and pressure on the front of the tank.
    c. Calculate the force and pressure on the valve.
    d. Calculate the work required to pump enough water out of the tank to decrease the pressure on the value by a half.

8. Find the surface area of the curve y = ln x for $0 \le x \le 1$ when rotated about the y-axis.

9. Solve $\frac{1}{y}\frac{dy}{dx} + \frac{m}{x} = \frac{\ln x}{y}$ where m is a constant.

10. Find the curve $y = f(x)$ that passes through the point (0,1) that has the property that the area under the curve is equal to the arc length of the curve on any interval [a,b].

11. A tank initially contains 20 L of water. A solution containing 1 g/L of a chemical flows into the tank at a rate of 3 L/min and the mixture flows out at a rate of 2 L/min. When does the concentration of tank reach 0.5 g/L?