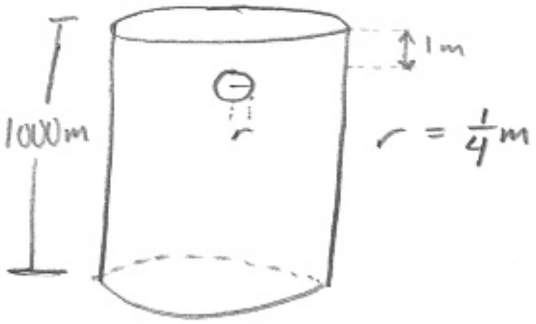
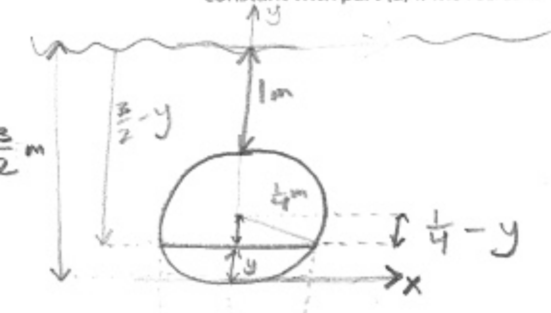
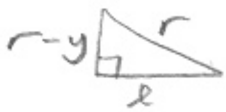
Zed Chance

1. A tank of water one thousand meters deep has a circular view window one meter below the surface of the water. If the view window has a radius of one-quarter of a meter...
   1. Find the hydrostatic pressure & force on the view window.
   2. At what depth would the window need to be moved to double the pressure on the window?
   3. What radius would the window need to have to keep the pressure constant with part (a) if moved to the depth you discovered in part (b)?







$\ell = \sqrt{r^2 - (r-y)^2}$

$= \sqrt{r^2 - (r^2 - 2ry + y^2)}$

$= \sqrt{2ry - y^2}$

$\ell = \sqrt{2(\frac{1}{4})y - y^2}$

$= \sqrt{\frac{1}{2}y - y^2}$ (2)

$W = 9800$

$F = 9800 \int_0^{\frac{1}{2}} (\frac{3}{2} - y)(2\sqrt{\frac{1}{2}y - y^2})\, dy$

$= 9800 \int_0^{\frac{1}{2}} 3\sqrt{\frac{1}{2}y - y^2} - 2y\sqrt{\frac{1}{2}y - y^2}\, dy$

$= 9800 \left[3\int_0^{\frac{1}{2}} \sqrt{\frac{1}{2}y - y^2}\, dy - 2\int_0^{\frac{1}{2}} y\sqrt{\frac{1}{2}y - y^2}\, dy \right]$

$-y^2 + \frac{1}{2}y + \frac{1}{16} - \frac{1}{16}$

$-(y - \frac{1}{4})^2 + \frac{1}{16}$

$= 9800 \left[3\int_0^{\frac{1}{2}} \sqrt{-(y-\frac{1}{4})^2 + \frac{1}{16}}\, dy - 2\int_0^{\frac{1}{2}} y\sqrt{-(y-\frac{1}{4})^2 + \frac{1}{16}}\, dy \right]$

$u = y - \frac{1}{4} \qquad du = dy$

$= 9800 \left[3\int_0^{\frac{1}{2}} \sqrt{-u^2 + \frac{1}{16}}\, du - 2\int_0^{\frac{1}{2}} (u + \frac{1}{4})\sqrt{-u^2 + \frac{1}{16}}\, du \right]$

$= 9800 \left[3\int_0^{\frac{1}{2}} \sqrt{\frac{-16u^2 + 1}{16}}\, du - 2\int_0^{\frac{1}{2}} (u + \frac{1}{4})\sqrt{\frac{-16u^2 + 1}{16}}\, du \right]$

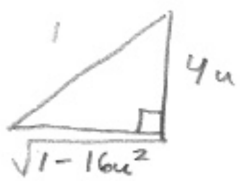
$= 9800 \left[\frac{3}{4}\int_0^{\frac{1}{2}} \sqrt{-16u^2 + 1}\, du - \frac{1}{2}\int_0^{\frac{1}{2}} (u + \frac{1}{4})\sqrt{-16u^2 + 1}\, du \right]$

$-16u^2 + 1$

$u_2 = \frac{1}{4}\sin\theta$

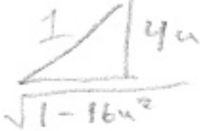
$du_2 = \frac{1}{4}\cos\theta\, d\theta$

$\theta = \sin^{-1}(\frac{4u}{1})$



$u_2 = \frac{1}{4}\sin\theta$ $\qquad\qquad$ $\theta = \sin^{-1}(4u)$

$du_2 = \frac{1}{4}\cos\theta \, d\theta$



$\cos\theta = \sqrt{1-16u^2}$

$$= 9800\left[\frac{3}{4}\int_0^{\frac{1}{2}} \cos\theta \, \frac{1}{4}\cos\theta\, d\theta - \frac{1}{2}\int_0^{\frac{1}{2}}\left(\frac{1}{4}\sin\theta + \frac{1}{4}\right)\cos\theta \, \frac{1}{4}\cos\theta \, d\theta\right]$$

$$= 9800\left[\frac{3}{16}\int_0^{\frac{1}{2}} \cos^2\theta \, d\theta - \frac{1}{32}\int_0^{\frac{1}{2}} \sin\theta\cos^2\theta + \cos^2\theta \, d\theta\right]$$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$2\cos^2\theta = 1 + \cos 2\theta$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$= 9800\left[\frac{3}{16}\int_0^{\frac{1}{2}} \frac{1}{2}(1 + \cos 2\theta)\, d\theta - \frac{1}{32}\int_0^{\frac{1}{2}} \cos^2\theta\,(\sin\theta + 1)\, d\theta\right]$$

$$= 9800\left[\frac{3}{32}\int_0^{\frac{1}{2}} 1 + \cos 2\theta \, d\theta - \frac{1}{64}\int_0^{\frac{1}{2}}(1 + \cos 2\theta)(\sin\theta + 1)\, d\theta\right]$$

$$= 9800\left[\frac{3}{32}\left(\theta + \frac{1}{2}\sin\theta\right)_0^{\frac{1}{2}} - \frac{1}{64}\int_0^{\frac{1}{2}} \sin\theta + 1 + \sin\theta\cos 2\theta + \cos 2\theta \, d\theta\right]$$

$$= 9800\left[\frac{3}{32}\left(\theta + \frac{1}{2}\sin\theta\right)_0^{\frac{1}{2}} - \frac{1}{64}\left[-\cos\theta + \theta + \frac{1}{2}\sin 2\theta + \frac{1}{2}\int_0^{\frac{1}{2}} \sin(\theta + 2\theta) + \sin(\theta - 2\theta) \, d\right.\right.$$

$\cos t \sin s = \frac{1}{2}\left(\sin(s+t) + \sin(s-t)\right)$

$$= 9800\left[\frac{3}{32}\left(\theta + \frac{1}{2}\sin\theta\right)_0^{\frac{1}{2}} - \frac{1}{64}\left[-\cos\theta + \theta + \frac{1}{2}\sin 2\theta - \frac{1}{3}\cos 3\theta - (-\cos\theta)\right]\right]_0^{\frac{1}{2}}$$

$$= 9800\left[\frac{3}{32}\left(\sin^{-1}(4u) + \frac{1}{2}(4u)\right) - \frac{1}{64}\left[\sin^{-1}(4u) + \frac{1}{2}\cos\left(3\sin^{-1}(4u)\right)\right]\right]_0^{\frac{1}{2}}$$

?

$u_1 = y - \frac{1}{4}$

$\therefore$

$$= 9800\left[\frac{3}{32}\left(\sin^{-1}(4y-1) + 2y - \frac{1}{2}\right) - \frac{1}{64}\left[\sin^{-1}(4y-1) + \frac{1}{2}\cos\left(3\sin^{-1}(4y-1)\right)\right]\right]_0^{\frac{1}{2}}$$

$$= 9800\left[\frac{3}{32}\left(\sin^{-1}(4y-1) + 2y - \frac{1}{2}\right) - \frac{1}{64}\left(\sin^{-1}(4y-1) + \frac{1}{2}\cos\left(3\sin^{-1}(4y-1)\right)\right)\right]_0^{\frac{1}{2}}$$

$$= 9800\left[\frac{3}{32}\left(\sin^{-1}(1) + \frac{1}{2}\right) - \frac{1}{64}\left(\sin^{-1}(1) + \frac{1}{2}\cos\left(3\sin^{-1}(1)\right)\right) - \left(\frac{3}{32}\left(\overset{\to 0}{\sin^{-1}(0)} - \frac{1}{2}\right) - \frac{1}{64}\left(\overset{\to 0}{\sin^{-1}(0)} + \frac{1}{2}\cos\left(\underbrace{3\sin^{-1}(0)}_{\frac{3\pi}{2}}\right)\right)\right)\right]$$

$$= 9800\left[\frac{3}{32}\left(\frac{\pi}{2} - \frac{1}{2}\right) - \frac{1}{64}\left(\frac{\pi}{2} - \frac{1}{2}\right) - \left(\frac{3}{32}\left(-\frac{1}{2}\right) - \frac{1}{64}\left(\frac{1}{2}(1)\right)\right)\right]$$

$$= 9800\left[\frac{3\pi}{64} - \frac{3}{64} - \frac{\pi}{128} + \frac{1}{128} - \left(\frac{-3}{64} - \frac{1}{64}\right)\right]$$

$$= 9800\left[\frac{5\pi}{128} + \frac{3}{128}\right]$$

$$\boxed{= 1225\left[\frac{5\pi + 3}{16}\right] N}$$

$\approx 1432 N$

At what depth does pressure double:

$$2\left(1225\left[\frac{5\pi + 3}{16}\right]\right) = 1225\left[\frac{5\pi + 3}{8}\right] N$$

$$1225\left[\frac{5\pi + 3}{8}\right] = 9800\int_0^{\frac{1}{2}} (\underset{\text{new height}}{h} - y)\left(2\sqrt{\tfrac{1}{2}y - y^2}\right)dy$$