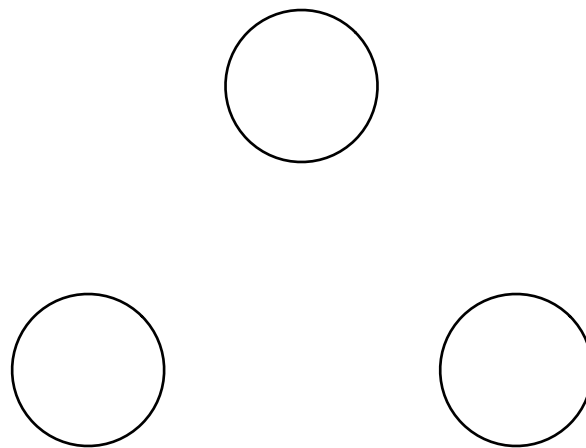
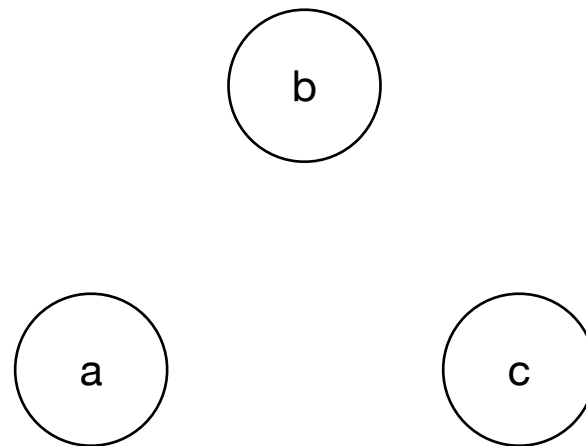


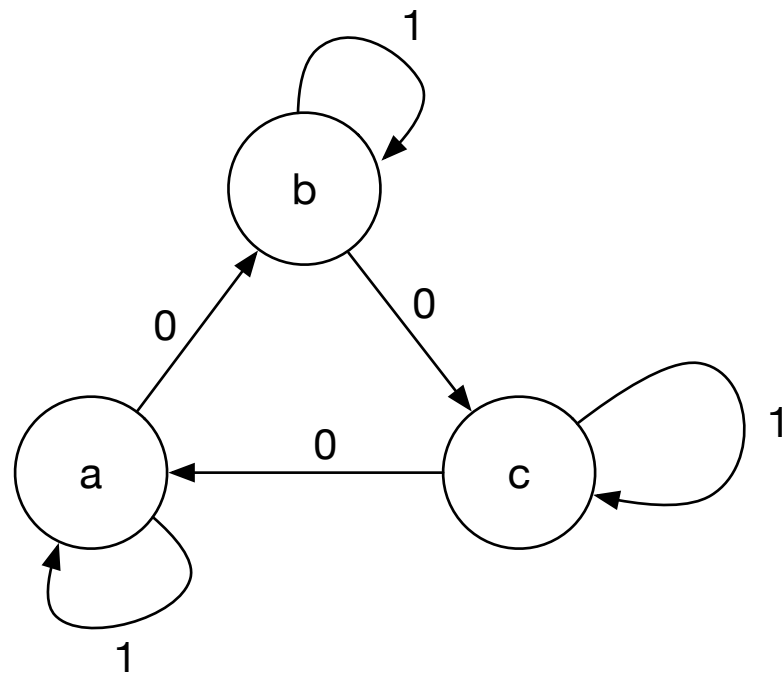
- A finite automata (FA) is a collection of states, represented as circles.



- A finite automata (FA) is a collection of states, represented as circles.
- If we wanted to represent the FA mathematically, we would need to label each state and put the labels in a set  $S = \{a, b, c\}$

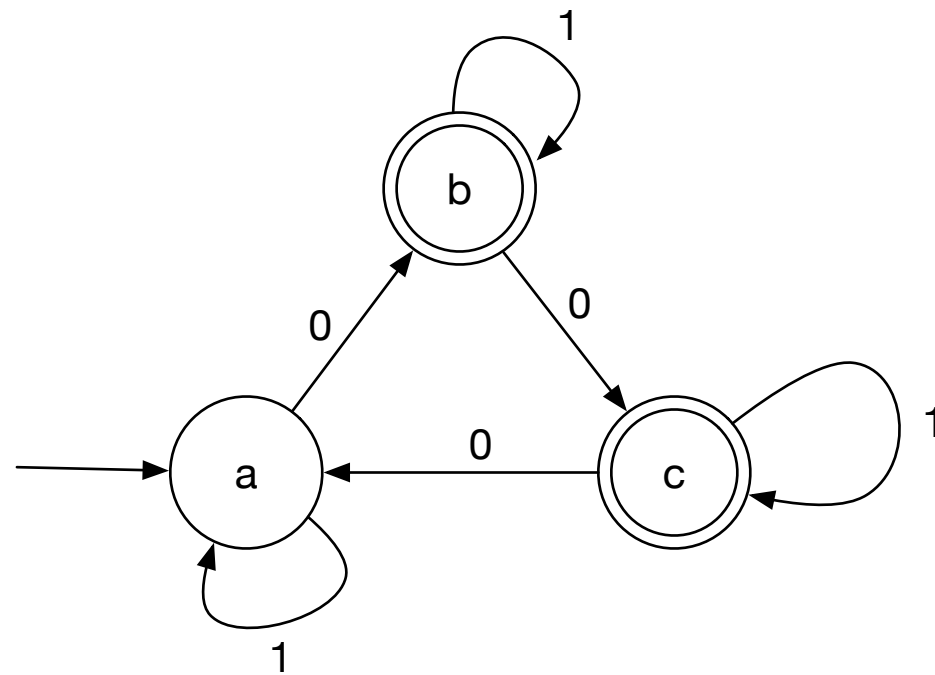


- Every state **must** have an arrow coming from it for every character in the input alphabet.
- Mathematically, we need to indicate the input alphabet  $A = \{0, 1\}$  and arrows. Since each arrow maps a (state, char) pair to a state, a function is a good representation  $F: S \times A \rightarrow S$ .

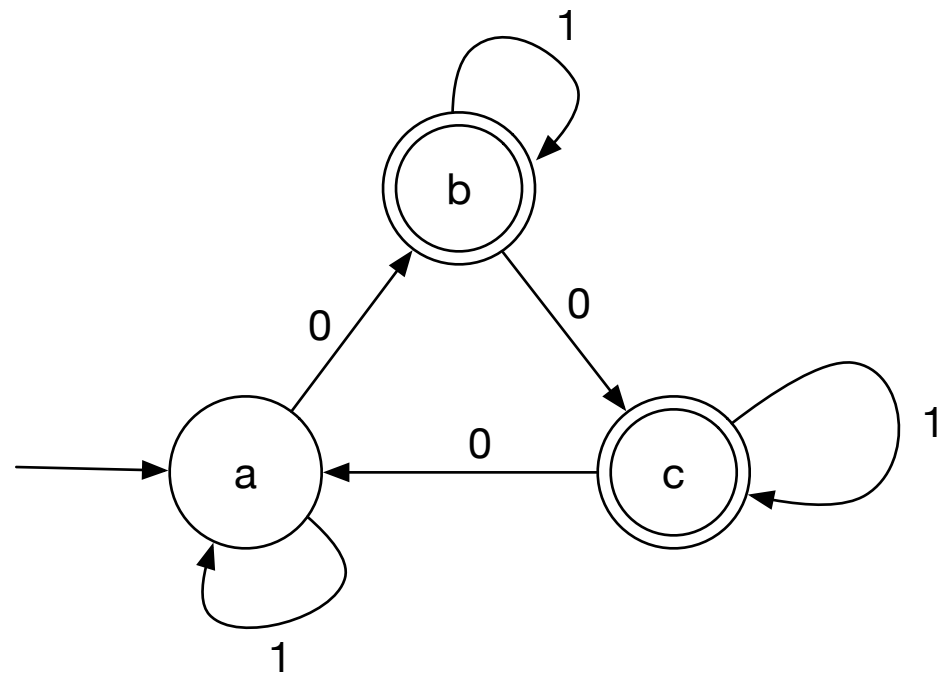


<b><i>F</i></b>	<b>0</b>	<b>1</b>
<b>a</b>	b	a
<b>b</b>	c	b
<b>c</b>	a	c

- There **must** be exactly one start state indicated by an unlabeled arrow from nowhere. And zero or more accept states (also called final) indicated by double circles.
- Mathematically, we can call  $S_i$  the start (initial) state,  $S_i = a$ , and let  $Y = \{b, c\}$  be the set of accept states.



- Because you can represent a FA graphically or mathematically, these are identical FA.



$$A = \{0, 1\}$$

$$S = \{a, b, c\}$$

$$S_i = a$$

$$Y = \{b, c\}$$

$F$  shown below in table form

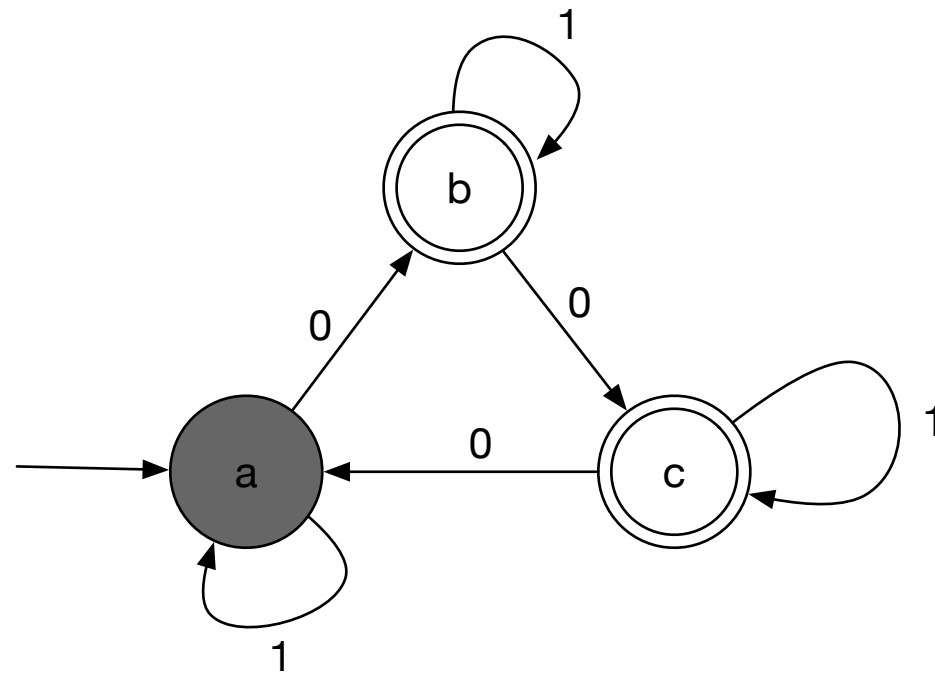
<b><math>F</math></b>	<b>0</b>	<b>1</b>
<b>a</b>	b	a
<b>b</b>	c	b
<b>c</b>	a	c

- Note about state labels: They can be any name you wish and are optional in drawings.

# FA Operation

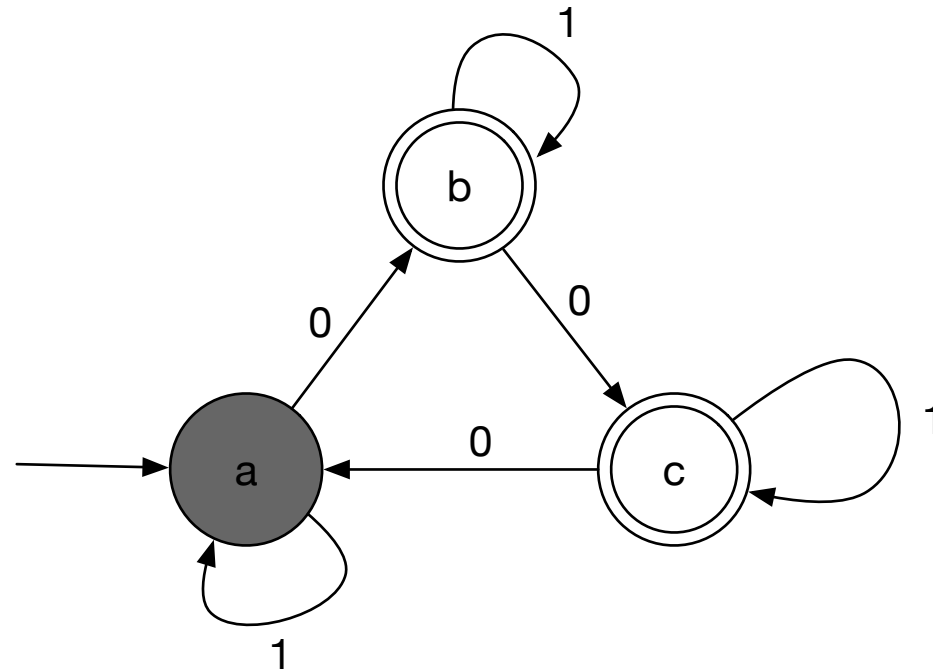
- When presented an input:
  - Start at start state
  - Consume characters from left to right
  - Follow arrow for each character consumed
  - If end in accept state "accept", else "reject"

- Example: Input is 10011. Start in the start state.



**Input: 10011**

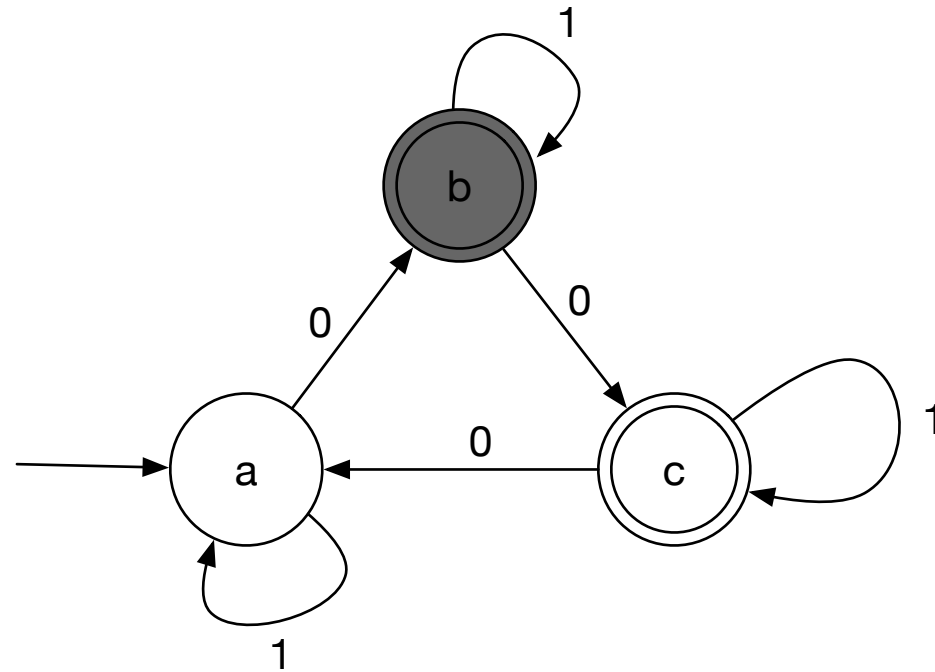
- Consume 1, follow arrow with 1, still in state "a"



**Remaining after consume and move: 0011**

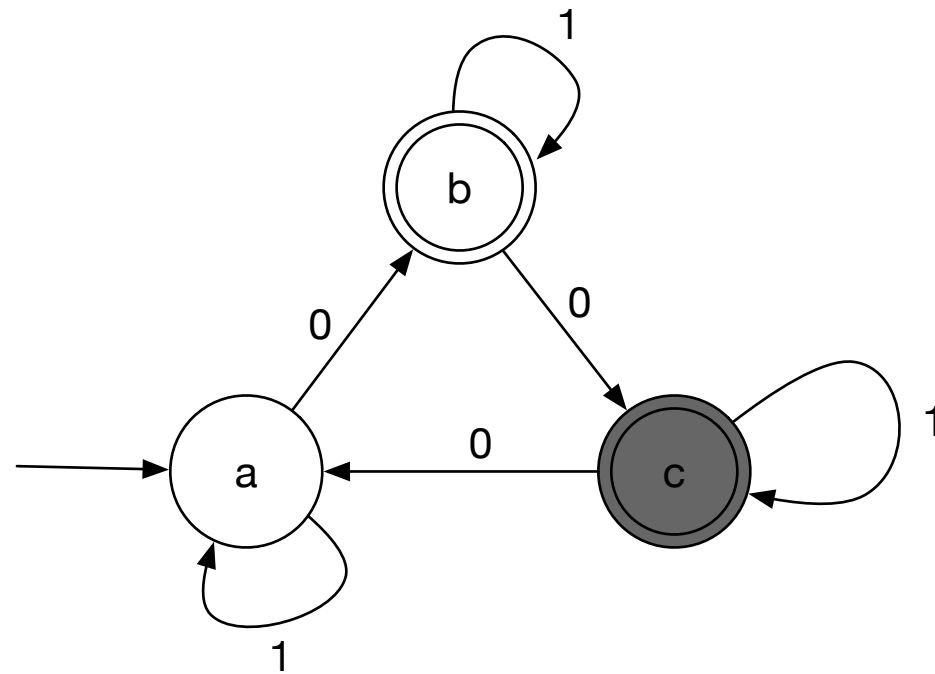


- Consume 0, follow arrow with 0, now in state "b"



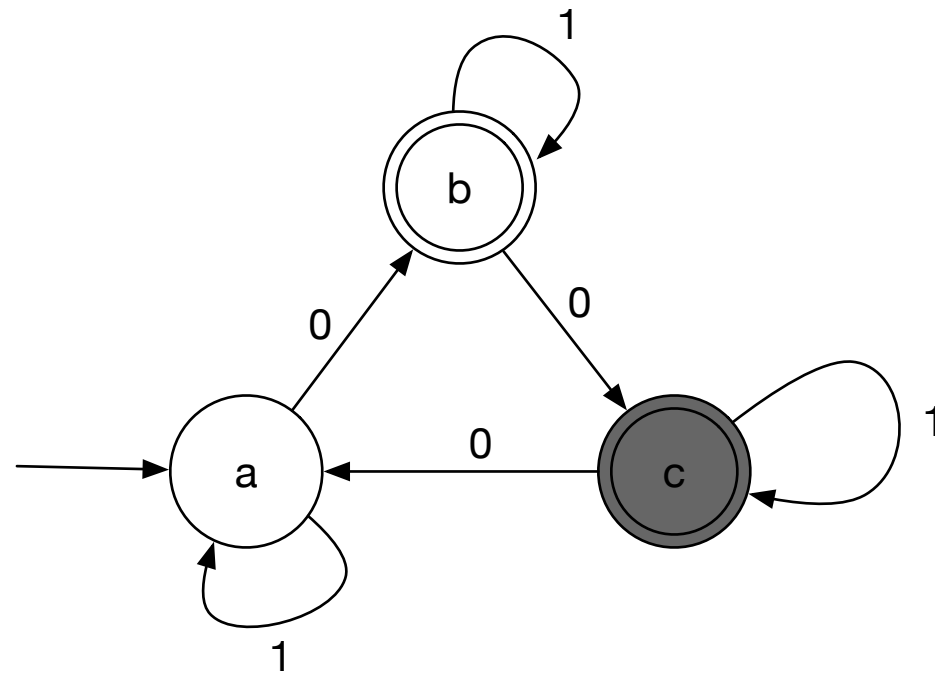
**Remaining after consume and move: 011**

- Consume 0, follow arrow with 0, now in state "c"



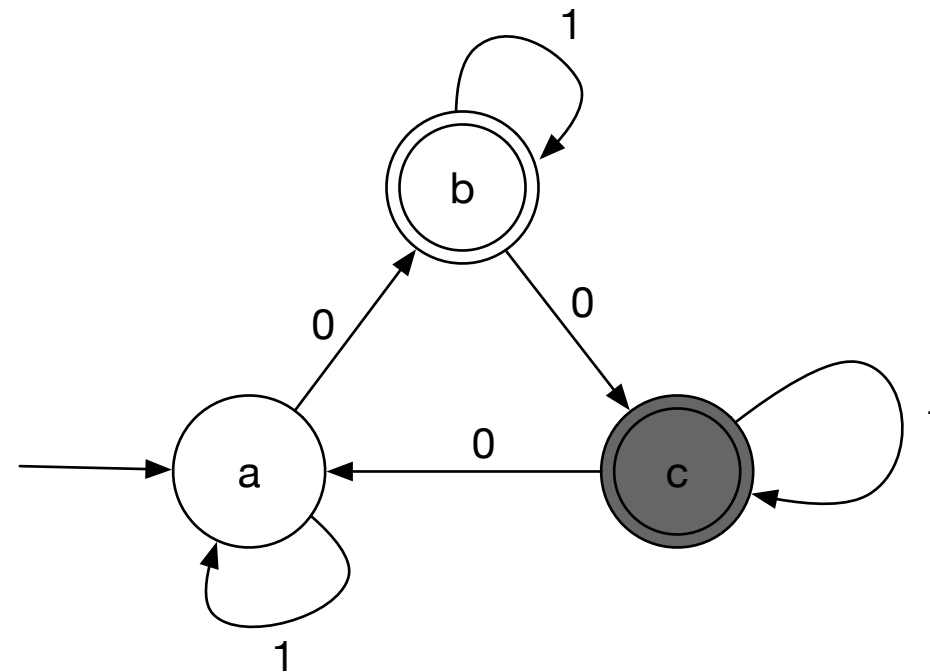
**Remaining after consume and move: 11**

- Consume 1, follow arrow with 1, still in state "c"



**Remaining after consume and move: 1**

- Consume 1, follow arrow with 1, still in state "c"



**Remaining after consume and move: (empty)**

- After consuming input, we end in an accept state.  
10011 is accepted by this FA.  
10011 is in the "language" of this machine.

# Meaning of FA

- You can often design an FA to accept strings that are easily described in English.
- This FA accepts all strings over alphabet  $\{0, 1\}$  that don't have a multiple of three 0's.
- If this machine is called  $M$ , then  $L(M) = \{ x \mid x \text{ is a string over alphabet } \{0, 1\} \text{ and the number of 0's in } x \text{ mod } 3 = 0 \}$

# Designing FA

- Give each state meaning. The only "memory" an FA has is the current state.
- Design the part of an FA that accepts good strings first.
- Make sure the FA is legal: one start state, arrow from each state for each character in the input alphabet.
- Test: try to find good string that's rejected; try to find bad string that's accepted. (This is how I grade FA's!)