Quiz 7
Zed Chance

Surface Area
Find the ~~length~~ of the curve for

$$x = \ln\left(\frac{1}{1-y}\right)$$

between 0 and ln(2). around x-axis

$$\frac{dx}{dy} = \frac{1}{1-y}$$

$$S = 2\pi \int_0^{\ln 2} y \sqrt{1 + \frac{1}{(1-y)^2}}\, dy$$

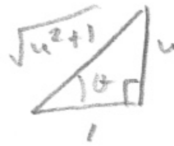
$$= 2\pi \int_0^{\ln 2} y \sqrt{\frac{(1-y)^2 + 1}{(1-y)^2}}\, dy$$

$$= 2\pi \int_0^{\ln 2} y \frac{\sqrt{(1-y)^2 + 1}}{1-y}\, dy$$

$u = 1 - y$
$du = -dy$
$y = 1 - u$

$$= -2\pi \int_0^{\ln 2} (1-u) \frac{\sqrt{u^2+1}}{u}\, du$$

$u = \tan\theta$
$du = \sec^2\theta\, d\theta$
$\theta = \tan^{-1}\left(\frac{u}{1}\right)$
$\sec\theta = \sqrt{u^2+1}$



$$= -2\pi \int_0^{\ln 2} (1 - \tan\theta) \frac{\sec\theta}{\tan\theta} \sec^2\theta\, d\theta$$

$$= -2\pi \int_0^{\ln 2} \frac{(1-\tan\theta)(\sec^3\theta)}{\tan\theta}\, d\theta$$

$$= -2\pi \int_0^{\ln 2} \frac{\sec^3\theta - \sec^3\theta \tan\theta}{\tan\theta}\, d\theta$$

$$= -2\pi \int_0^{\ln 2} \frac{\sec^3\theta}{\tan\theta} - \frac{\sec^3\theta \tan\theta}{\tan\theta}\, d\theta$$

$$= -2\pi \int_0^{\ln 2} \frac{\sec^3\theta}{\tan\theta} - \sec^3\theta\, d\theta$$

$$\frac{\sec^3\theta}{\tan\theta} = \frac{\cos\theta}{\cos^3\theta \sin\theta} = \frac{1}{\cos^2\theta \sin\theta}$$

$$= -2\pi \int_0^{\ln 2} \frac{1}{\cos^2\theta \sin\theta} - \frac{1}{\cos^3\theta}\, d\theta$$

$$= -2\pi \left[\int_0^{\ln 2} \frac{\sec^2\theta}{\sin\theta}\, d\theta - \int_0^{\ln 2} \sec^3\theta\, d\theta\right]$$

$\cos^2\theta + \sin^2\theta = 1$
$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$$= -2\pi\left[\underbrace{\int_0^{\ln 2}\frac{\sec^2\theta}{\sin\theta}\,d\theta}_{I_1} - \underbrace{\int_0^{\ln 2}\sec^3\theta\, d\theta}_{I_2}\right]$$

$$I_1 = \int \frac{\sec^2\theta}{\sin\theta}\,d\theta$$

$u_2 = \sin\theta$
$du_2 = \cos\theta\, d\theta$

$dv = \sec^2\theta\, d\theta$
$v = \tan\theta$

$$= \sin\theta\tan\theta - \int \tan\theta\cos\theta\, d\theta$$

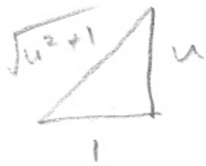
$$= \sin\theta\tan\theta - \int \frac{\sin\theta}{\cos^2\theta}\,d\theta$$

$w = \cos\theta$
$dw = -\sin\theta\, d\theta$

$$= \sin\theta\tan\theta + \int \frac{1}{w^2}\,dw$$

$\int w^{-2} = -\frac{1}{w}$

$\theta = \tan^{-1}\left(\frac{u}{1}\right)$



$$= \sin\theta\tan\theta - \frac{1}{w}$$

$$= \sin\theta\tan\theta - \sec\theta$$

$$= \frac{u^2}{\sqrt{u^2+1}} - \sqrt{u^2+1}\,\Bigg|_0^{\ln 2}$$

$u = 1 - y$

$$= \frac{(1-y)^2}{\sqrt{(1-y)^2+1}} - \sqrt{(1-y)^2+1}\,\Bigg|_0^{\ln 2}$$

$$I_2 = \int \sec^3\theta\, d\theta$$

$\cos^2\theta + \sin^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$

$$= \int \sec^2\theta\sec\theta\, d\theta$$

$u_3 = \sec\theta$
$du_3 = \sec\theta\tan\theta\, d\theta$

$dv_2 = \sec^2\theta\, d\theta$
$v_2 = \tan\theta$

$$= \sec\theta\tan\theta - \int \tan\theta\sec\theta\tan\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)(\sec\theta)\, d\theta$$

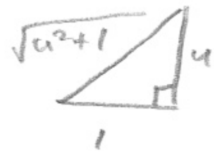
$$= \sec\theta\tan\theta - \underbrace{\int \sec^3\theta\, d\theta}_{I_2} - \int \sec\theta\, d\theta$$

$$= \sec\theta\tan\theta - \underbrace{\int \sec^3\theta \, d\theta}_{I_2} - \int \sec\theta \, d\theta$$

$$2I_2 = \sec\theta\tan\theta - \int \sec\theta \, d\theta$$

$$2I_2 = \sec\theta\tan\theta - \ln\left|\sec\theta + \tan\theta\right|$$

$\theta = \tan^{-1}\left(\frac{u}{1}\right)$



$u = 1 - y$

$$= \frac{1}{2}\left[\sec\theta\tan\theta - \ln\left|\sec\theta + \tan\theta\right|\right]$$

$$= \frac{1}{2}\left[\sqrt{u^2+1}\cdot u - \ln\left|\sqrt{u^2+1} + u\right|\right]$$

$$= \frac{1}{2}\left[\sqrt{(1-y)^2+1}\,(1-y) - \ln\left|\sqrt{(1-y)^2+1} + (1-y)\right|\right]_0^{\ln 2}$$

$$\therefore = -2\pi\left[I_1 - I_2\right]_0^{\ln 2}$$

$$= -2\pi\left[\frac{(1-y)^2}{\sqrt{(1-y)^2+1}} - \sqrt{(1-y)^2+1} - \frac{1}{2}\left[\sqrt{(1-y)^2+1}\,(1-y) - \ln\left|\sqrt{(1-y)^2+1} + (1-y)\right|\right]\right]_0^{\ln 2}$$

$$\boxed{= -2\pi\left[\frac{(1-\ln 2)^2}{\sqrt{(1-\ln 2)^2+1}} - \sqrt{(1-\ln 2)^2+1} - \frac{1}{2}\left[\sqrt{(1-\ln 2)^2+1}\,(1-\ln 2) - \ln\left|\sqrt{(1-\ln 2)^2+1} + (1-\ln 2)\right|\right] - \left(\frac{1}{\sqrt{2}} - \sqrt{2} - \frac{1}{2}\left[\sqrt{2}\,(1) - \ln\left|\sqrt{2} + 1\right|\right]\right)\right]}$$