Math 31 | Chapter 9 Assignment | Differential Equations

Due: April 2^{nd} by 11:59PM

Instructions

- Complete the follow exercises on sperate sheets of paper. Scan your solutions and upload a PDF document. The file should have the following naming convention:

"Last Name First Name Assignment Name.pdf"

"Albright Charles Chapter 9 Assignment.pdf"

- Make sure your pages are numbered in the lower right-hand corner.
- Make sure each page has your full name and the name of the assignment in the upper right-hand corner of each page.
- **Note:** You do not need to include this page in your solutions.

Solutions

- Because of the unique circumstances of our situation, take special care with your solutions. Make sure they are complete, organized, clear and thorough. Error on the side explaining too much.
- Your final answer should be simplified and exact.
- Graphs should be clear, legible and labeled.

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1. Verify $y(x) = c_1 \sin x + c_2 \cos x - (\cos x)\ln(\sec x + \tan x)$ is a solution to $y'' + y = \tan x$.
2. Find a function f such that $f'(x) = xf(x) - x$ and $f(0) = 2$.
3. Solve the DE $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right)$ if k, M and m are constants.
4. Solve $y' + \frac{2}{x}y = \frac{y^3}{x^2}$ using the following technique:

   A Bernoulli differential equation (named after James Bernoulli) is of the form

   $$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

   Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, show that the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

   $$\frac{du}{dx} + (1 - n)P(x)u = (1 - n)Q(x)$$

5. Solve the second-order equation $xy'' + 2y' = 12x^2$ by making the substitution $u = y'$.
6. A tank with a capacity of 400 L is full of a mixture of water and chlorine with a concentration of 0.05 g of chlorine per liter. In order to reduce the concentration of chlorine, fresh water is pumped into the tank at a rate of 4 L/s. The mixture is kept stirred and is pumped out at a rate of 10 L/s. Find the amount of chlorine in the tank as a function of time.
7. A tank with a capacity of 1000 L is fill with 800 L of a mixture of water and chlorine with a concentration of 0.1 g of chlorine per liter. A solution of 0.075 g per liter is pumped into the tank at a rate of 5 L/s. The mixture is kept stirred and is pumped out at a varying rate $\left(5 - \frac{2}{t+1}\right)\frac{L}{s}$ where t is seconds. Find the amount of chlorine in the tank when the amount of water is at it's maximum.