Calculate the following integrals: (20 points EACH)

1. $\int_{-2}^{3} \frac{x}{x+1}\,dx$

LIMIT SWITCH

ASIDE

$$\int \frac{x}{x+1}\,dx$$

$$= \int \frac{x+1-1}{x+1}\,dx$$

$$= x - \ln|x+1| + C$$

$\boxed{8\text{PTS}}$

$$\int_{-2}^{3} \frac{x}{x+1}\,dx$$

$$= \lim_{t \to -1^-} \int_{-2}^{t} \frac{x}{x+1}\,dx + \lim_{s \to -1^+} \int_{s}^{3} \frac{x}{x+1}\,dx$$

$$= \lim_{t \to -1^-} \left(x - \ln|x+1|\right)\Big|_{-2}^{t} + \lim_{s \to -1^+} \left(x - \ln|x+1|\right)\Big|_{s}^{3}$$

$$= \lim_{t \to -1^-} \left(t - \ln|t+1| - (-2) + \ln|-2+1|\right)$$

$\boxed{12\text{PTS}}$

$$+ \lim_{s \to -1^+} \left(3 - \ln|3+1| - s + \ln|s+1|\right)$$

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$\boxed{\text{DIVERGENT}}$

2. $\int x \arccos x \, dx$

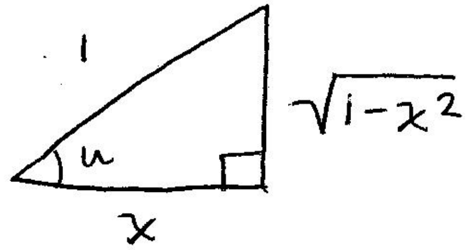
$u = \arccos(x) \qquad dv = x\,dx$

$\cos(u) = x \qquad v = \frac{1}{2}x^2$

$-\sin(u)\frac{du}{dx} = 1$

$du = -\csc(u)\,dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$



STARTING
IBP
6 PTS

$uv - \int v\,du$

$= \frac{1}{2}x^2\arccos(x) - \int\left(-\frac{1}{2}\right)\frac{x^2}{\sqrt{1-x^2}}\,dx$

$= \frac{1}{2}x^2\arccos(x) + \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx$

$= \frac{1}{2}x^2\arccos(x) + \frac{1}{2}\int\frac{\cos^2\theta(-\sin\theta)}{\sin\theta}\,dx$

$= \frac{1}{2}x^2\arccos(x) - \frac{1}{2}\int\cos^2\theta\,d\theta$

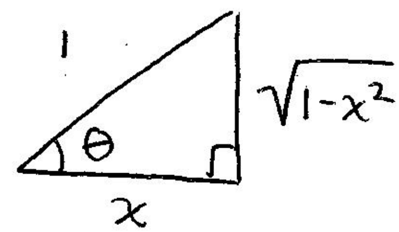
$= \frac{1}{2}x^2\arccos(x) - \frac{1}{2}\int\frac{1+\cos 2\theta}{2}\,d\theta$

$= \frac{1}{2}x^2\arccos(x) - \frac{1}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

$= \frac{1}{2}x^2\arccos(x) - \frac{1}{4}\cos^{-1}(x) - \frac{1}{4}\left(x\sqrt{1-x^2}\right) + C$

$x = \cos\theta$

$\cos^{-1}(x) = \theta$



$dx = -\sin\theta\,d\theta$

3. $\int \frac{x}{(x^2-5x+6)^2}\,dx$

PFD
10 PTS

$$= \int \frac{x}{(x-2)^2(x-3)^2}\,dx$$

$$= \int \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)} + \frac{D}{(x-3)^2}\,dx = (*)$$

$$x = A(x-2)(x-3)^2 + B(x-3)^2 + C(x-3)(x-2)^2 + D(x-2)^2$$

$\boxed{x=2}$

$2 = B(-1)^2$

$\therefore B = 2$

$\boxed{x=3}$

$3 = D(1)^2$

$\therefore D = 3$

$\boxed{x^3}$

$0 = A + C$

$\boxed{x=0}$

$0 = -18A + 9B - 12C + 4D$

$0 = -18A + 18 + 12A + 12$

$0 = -6A + 30$

$\therefore A = 5$

$\therefore C = -5$

$(*)$

$$= \int \frac{5}{x-2} + \frac{2}{(x-2)^2} - \frac{5}{(x-3)} + \frac{3}{(x-3)^2}\,dx$$

$$= 5\ln\left|\frac{x-2}{x-3}\right| - \frac{2}{(x-2)} - \frac{3}{(x-3)} + C$$

4. $\int_0^1 \frac{\ln(x)}{\sqrt{x}}\,dx$

$$= \lim_{t\to 0^+} \int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx = (*)$$

$u = \ln x \qquad dv = x^{-\frac{1}{2}}\,dx$

$du = \frac{1}{x}\,dx \qquad v = 2x^{\frac{1}{2}}$

$uv - \int v\,du$

$= 2\sqrt{x}\ln x - 2\int x^{-\frac{1}{2}}\,dx$

$= 2\sqrt{x}\ln x - 2\left(2x^{\frac{1}{2}}\right) + C$

$= 2\sqrt{x}\ln x - 4\sqrt{x} + C$

ASIDE

$\lim_{t\to 0^+} \frac{\ln(t)}{\left(\frac{1}{\sqrt{t}}\right)} \overset{LH}{=} \lim_{t\to 0^+} \frac{\left(\frac{1}{t}\right)}{-\frac{1}{2}t^{-\frac{3}{2}}}$

$= \lim_{t\to 0^+} (-2)t^{-1}t^{\frac{3}{2}}$

$= -2\lim_{t\to 0^+} t^{\frac{1}{2}} = 0$

NO LIMIT

-12

NOT SHOWING

-5

$(*) = \lim_{t\to 0^+} \int_t^1 \frac{\ln(x)}{\sqrt{x}}\,dx$

$= \lim_{t\to 0^+} \left(2\sqrt{x}\ln x - 4\sqrt{x}\right)\Big|_t^1$

$= \lim_{t\to 0^+} \left(2(0) - 4(1) - 2\sqrt{t}\ln(t) - 4\sqrt{t}\right)$

$= -4 - 2\lim_{t\to 0^+} \sqrt{t}\ln(t) = \boxed{-4}$

LIMIT / INTEGRAL
10 PTS 10PTS

5. $\int_{-\infty}^{\infty} \frac{e^x}{e^{4x}+5e^{2x}+4}\,dx = (*)$

$u = e^x$ $\quad x \to \infty$ $\quad x \to -\infty$

$du = e^x\,dx$ $\quad e^x \to \infty$ $\quad e^x \to 0$

$$(*) = \int_0^\infty \frac{1}{u^4+5u^2+4}\,du$$

IF THIS TRICK NOT DONE TWO LIMITS! (5PTS)

$$= \int_0^\infty \frac{Au+B}{u^2+4} + \frac{Cu+D}{u^2+1}\,du$$

$$1 = (Au+B)(u^2+1) + (Cu+D)(u^2+4)$$

$$1 = Au^3 + Au + Bu^2 + B$$
$$Cu^3 + 4Cu + Du^2 + 4D$$

$$1 = (A+C)u^3 + (A+4C)u + (B+D)u^2 + (B+4D)$$

$0 = A+C$ $\quad 0 = B+D$

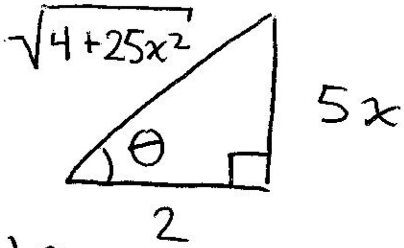
$0 = A+4C$ $\quad 1 = B+4D$ $\quad \therefore D = \frac{1}{3}$

$\therefore 0 = A = C$ $\quad 1 = 3D$ $\quad \therefore B = -\frac{1}{3}$

$$(*) = \lim_{t\to\infty} \int_0^t -\frac{1}{3}\frac{1}{u^2+4} + \frac{1}{3}\frac{1}{u^2+1}\,du$$

$$= \lim_{t\to\infty}\left[-\frac{1}{3}\cdot\frac{1}{2}\tan^{-1}(u) + \frac{1}{3}\tan^{-1}(u)\right]\Bigg|_0^t = \lim_{t\to\infty}\frac{1}{6}\tan^{-1}(u) = \boxed{\frac{\pi}{12}}$$

6. $\int \frac{1}{\sqrt{4+25x^2}}\,dx$



$x = \frac{2}{5}\tan\theta$

$x^2 = \frac{4}{25}\tan^2\theta$

$4+25x^2 = 4+4\tan^2\theta$

$= 4(\sec^2\theta)$

$x = \frac{2}{5}\tan\theta$

$dx = \frac{2}{5}\sec^2\theta\,d\theta$

$\frac{5x}{2} = \tan\theta$

$$(*) = \int \frac{\frac{2}{5}\sec^2\theta}{\sqrt{4\sec^2\theta}}\,d\theta$$

$$= \frac{2}{5}\cdot\frac{1}{2}\int \sec\theta\,d\theta$$

$$= \frac{1}{5}\ln\left|\sec\theta+\tan\theta\right| + c$$

$$= \frac{1}{5}\ln\left|\frac{\sqrt{4+25x^2}}{2} + \frac{5x}{2}\right| + C$$